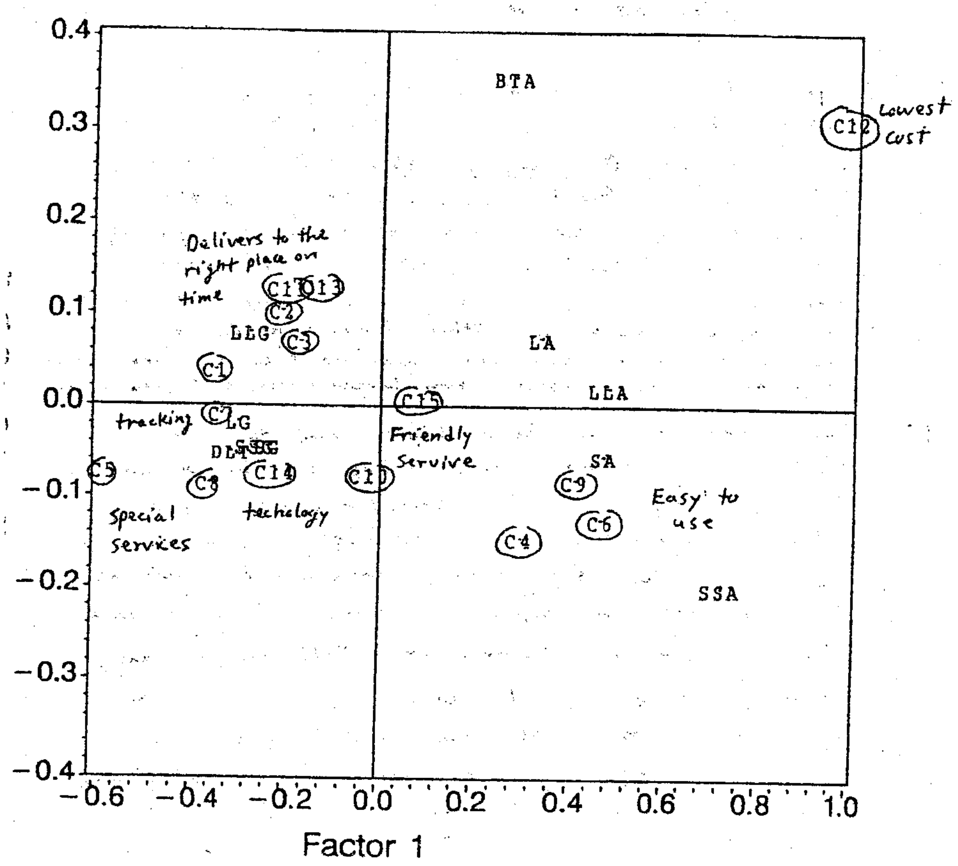
6.(B)



9.1

$$L' = [.9 \quad .7 \quad .5]; \qquad LL' = \begin{bmatrix} .81 & .63 & .45 \\ .63 & .49 & .35 \\ .45 & .35 & .25 \end{bmatrix}$$

so $\rho = LL' + \Psi$

9.2 a) For $m = 1$

$$h_1^2 = \ell_{11}^2 = .81$$
$$h_2^2 = \ell_{21}^2 = .49$$
$$h_3^2 = \ell_{31}^2 = .25$$

The communalities are those parts of the variances of the variables explained by the single factor.

b) $\text{Corr}(Z_i, F_1) = \text{Cov}(Z_i, F_1)$, $i = 1,2,3$. By (9-5) $\text{Cov}(Z_i, F_1) = \ell_{i1}$. Thus $\text{Corr}(Z_1, F_1) = \ell_{11} = .9$; $\text{Corr}(Z_2, F_1) = \ell_{21} = .7$; $\text{Corr}(Z_3, F_1) = \ell_{31} = .5$. The first variable, Z_1, has the largest correlation with the factor and therefore will probably carry the most weight in naming the factor.

9.3 a) $L = \sqrt{\lambda_1}\, \underset{\sim}{e}_1 = \sqrt{1.96}\begin{bmatrix} .625 \\ .593 \\ .507 \end{bmatrix} = \begin{bmatrix} .876 \\ .831 \\ .711 \end{bmatrix}$. Slightly different from result in Exercise 9.1.

b) Proportion of total variance explained $= \frac{\lambda_1}{p} = \frac{1.96}{3} = .65$

9.4

$$\underset{\sim}{\rho} = \underset{\sim}{\rho} - \Psi = LL' = \begin{bmatrix} .81 & .63 & .45 \\ .63 & .49 & .35 \\ .45 & .35 & .25 \end{bmatrix}$$

$$L = \sqrt{\lambda_1}\, \underset{\sim}{e}_1 = \sqrt{1.55}\begin{bmatrix} .7229 \\ .5623 \\ .4016 \end{bmatrix} = \begin{bmatrix} .9 \\ .7 \\ .5 \end{bmatrix}$$

Result is consistent with results in Exercise 9.1. It should be since $m = 1$ common factor completely determines $\tilde{\underset{\sim}{\rho}} = \underset{\sim}{\rho} - \Psi$.

9.7 From the equation $\Sigma = LL' + \Psi$, $m = 1$, we have

$$\begin{bmatrix} \sigma_{11} & \sigma_{12} \\ \sigma_{12} & \sigma_{22} \end{bmatrix} = \begin{bmatrix} \ell_{11}^2 + \psi_1 & \ell_{11}\ell_{21} \\ \ell_{11}\ell_{21} & \ell_{21}^2 + \psi_2 \end{bmatrix}$$

so $\sigma_{11} = \ell_{11}^2 + \psi_1$, $\sigma_{22} = \ell_{21}^2 + \psi_2$ and $\sigma_{12} = \ell_{11}\ell_{21}$.
Let $\rho = \sigma_{12}/\sqrt{\sigma_{11}}\sqrt{\sigma_{22}}$. Then, for any choice $|\rho|\sqrt{\sigma_{22}} \le \ell_{21} \le \sqrt{\sigma_{22}}$, set $\ell_{11} = \sigma_{12}/\ell_{21}$ and check $\sigma_{12} = \ell_{11}\ell_{21}$. We obtain $\psi_1 = \sigma_{11} - \ell_{11}^2 = \sigma_{11} - \frac{\sigma_{12}^2}{\ell_{21}^2} \ge \sigma_{11} - \frac{\sigma_{12}^2}{\rho^2\sigma_{22}} = \sigma_{11} - \sigma_{11} = 0$ and $\psi_2 = \sigma_{22} - \ell_{21}^2 \ge \sigma_{22} - \sigma_{22} = 0$. Since ℓ_{21} was arbitrary within a suitable interval, there are an infinite number of solutions to the factorization.

9.8 $\Sigma = LL' + \Psi$ for $m = 1$ implies

$$\begin{pmatrix} 1 = \ell_{11}^2 + \psi_1 & .4 = \ell_{11}\ell_{21} & .9 = \ell_{11}\ell_{31} \\ & 1 = \ell_{21}^2 + \psi_2 & .7 = \ell_{21}\ell_{31} \\ & & 1 = \ell_{31}^2 + \psi_3 \end{pmatrix}$$

Now $\frac{\ell_{11}}{\ell_{21}} = \frac{.9}{.7}$ and $\ell_{11}\ell_{21} = .4$, so $\ell_{11}^2 = (\frac{.9}{.7})(.4)$ and $\ell_{11} = \pm .717$. Thus $\ell_{21} = \pm .558$. Finally, from $.9 = \ell_{11}\ell_{31}$, we have $\ell_{31} = \pm .9/.717 = \pm 1.255$.

Note all the loadings must be of the same sign because all the covariances are positive. We have

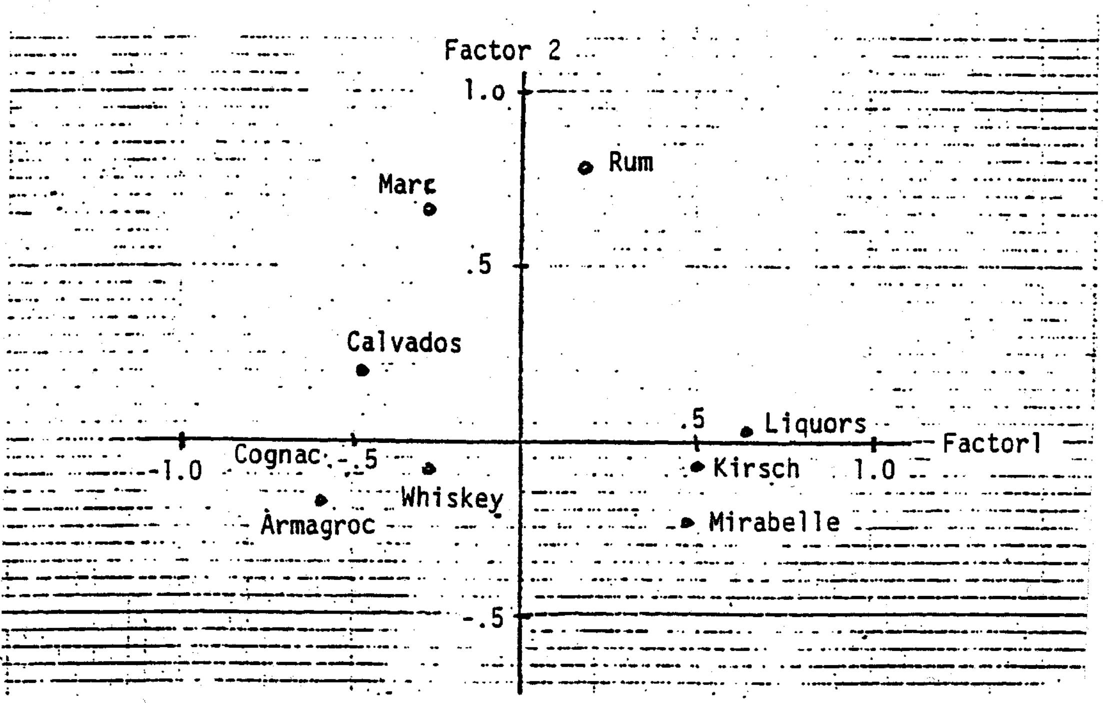
$$LL' = \begin{bmatrix} .717 \\ .558 \\ 1.255 \end{bmatrix} [.717 \ .558 \ 1.255] = \begin{bmatrix} .514 & .4 & .9 \\ .4 & .3111 & .7 \\ .9 & .7 & 1.575 \end{bmatrix}$$

so $\psi_3 = 1 - 1.575 = -.575$, which is inadmissible as a variance.

9.9

(a) Stoetzel's interpretation seems reasonable. The first factor seems to contrast sweet with strong liquors.

(b)



It doesn't appear as if rotation of the factor axes is necessary.

Variable	Specific Variance	Communality
Skull length	.5976	.4024
Skull breadth	.7582	.2418
Femur length	.1221	.8779
Tibia length	.0000	1.0000
Humerus length	.0095	.9905
Ulna length	.0938	.9062

(c) The proportion of variance explained by each factor is:

Factor 1 : $\frac{1}{6}\sum_{i=1}^{p} \ell_{1i}^2 = \frac{4.0001}{6}$ or 66.7%

Factor 2 : $\frac{1}{6}\sum_{i=1}^{p} \ell_{2i}^2 = \frac{.4177}{6}$ or 6.7%

(d) $R - \hat{L}_z \hat{L}_z' - \hat{\Psi} =$

$$\begin{bmatrix} 0 \\ .193 & 0 \\ -.017 & -.032 & 0 \\ .000 & .000 & .000 & 0 \\ -.000 & .001 & .000 & .000 & 0 \\ -.001 & -.018 & .003 & .000 & .000 & 0 \end{bmatrix}$$