Stat 501Solutions and Comments for Assignment 5Spring 2005

1. (a) After a varimax rotation, the first rotated component is a wealth

component, the second rotated component reflects the proportion of families

with

young children who live in smaller owner occupied homes, and third component is an unemployment component.

1 (b)

- 0.62430	- 0.19180	0.01276
0.86741	0.11404	- 0.19289
0.97744	0.00719	0.00405
- 0.92934	0.01074	- 0.16651
- 0.92795	0.49573	-0.11642
0.01287	0.17323	0.96971
0.68394	0.32706	- 0.22777
0.05446	0.93394	0.17000

The interpretation of first 3 principal components is similar to 1 (a).

The correlations among the factor scores

 $\left[\begin{array}{cccc} 1.00000 & 0.12365 & -0.21037 \\ 0.12365 & 1.00000 & -0.06829 \\ -0.21037 & -0.06829 & 1.00000 \end{array}\right]$

2. (a)	specific variances:	.597	.758	.122	.000	.010	.094
	communalities:	.403	.242	.878	1.000	.990	.906

(b) Factor 1 is an overall size component that concentrates on the wing size measurements. It accounts for 38.4% of the variation in the standardized traits.

Factor 2 is an overall size component that concentrates of the leg measurements. It accounts for 35.3% of the variation in the standardized traits.

(c) All traits have moderate to large positive correlations. Consequently, all of

the

traits have positive loadings on the first two factors. Since the wing measurements (X_5 and X_6) have the strongest correlation, they are featured in the

first factor. The leg measurements $(X_3 \text{ and } X_4)$ are almost as strongly correlated as the wing measurements, and they are featured in the second factor.

(d) Only the correlation between the two skull measurements is not well explained by the loadings on the first two factors.

0	0.1924	- 0.0175	- 0.0006	- 0.0008	- 0.0012
0.1924	0	- 0.0328	- 0.0004	- 0.0001	- 0.0185
-0.0175	- 0.0328	0	0.0000	0.0000	0.0035
-0.0006	- 0.0004	0.0000	0	0.0000	- 0.0000
-0.0008	- 0.0001	0.0000	0.0000	0	0.0000
-0.0012	- 0.0185	0.0035	- 0.0000	0.0000	0

(e) The combined amount of variation accounted for by two factors would be the same for rotated or unrotated factors. Of course, the individual rotated factors would not have the same interpretation as the two unrotated factors. The communalities and specific variances in part (a) are the same for two rotated factors and the original pair of unrotated factors. The residual matrix in part (d) would be the same for both sets of factors.

(f)



(b)
$$\begin{array}{c} 1 & \rho & \rho & \rho \\ \rho & 1 & \rho & \rho \\ \rho & \rho & 1 & \rho \\ \rho & \rho & \rho & 1 \end{array} \begin{bmatrix} 0.5 \\ 0.5 \\ 0.5 \\ 0.5 \\ 0.5 \end{bmatrix} = (3\rho+1) \begin{bmatrix} 0.5 \\ 0.5 \\ 0.5 \\ 0.5 \\ 0.5 \end{bmatrix}$$
 and $\begin{array}{c} \hat{\lambda}_1 = 3\rho+1 \\ \hat{\lambda}_1 = 3\rho+1 \\ 0.5 \\ 0.5 \end{bmatrix}$
(b) $\begin{array}{c} \hat{\lambda}_2 = \hat{\lambda}_3 = \hat{\lambda}_4 = 1-\rho \end{array}$

(c) Defining the vector of loadings and the diagonal matrix of specific variances as

$$L = \begin{bmatrix} \sqrt{\rho} \\ \sqrt{\rho} \\ \sqrt{\rho} \\ \sqrt{\rho} \end{bmatrix} \text{ and } \Psi = (1 - \rho)I, \text{ we have } \begin{bmatrix} 1 & \rho & \rho & \rho \\ \rho & 1 & \rho & \rho \\ \rho & \rho & 1 & \rho \\ \rho & \rho & \rho & 1 \end{bmatrix} = LL' + \Psi$$

- (d) No, the correlations will not be equal unless each trait has the same loading on the single factor. Problem 4 illustrates a situation where there is only one factor and the correlations are not all equal.
- 4. (a) Assume the model $X_i = a_i F + \epsilon_i$ for i 1, 2, ..., p traits, where

$$\begin{split} E(F) &= \mu_F \quad Var(F) = \sigma_F^2 \qquad E(\epsilon_i) = 0 \quad Var((\epsilon_i) = \psi_i \\ Cov(\epsilon_i, \epsilon_j) &= 0 \quad \text{and} \quad Cov(\epsilon_i, F) = 0 \quad \text{for all } i=1, 2, ..., p. \end{split}$$

Then

$$\mathbf{X} = \begin{bmatrix} \mathbf{X}_1 \\ \vdots \\ \mathbf{X}_p \end{bmatrix} = \begin{bmatrix} \mathbf{a}_1 \\ \vdots \\ \mathbf{a}_p \end{bmatrix} \mathbf{F} + \begin{bmatrix} \boldsymbol{\varepsilon}_1 \\ \vdots \\ \boldsymbol{\varepsilon}_p \end{bmatrix}$$

where

$$\mathbf{E}\begin{bmatrix} \boldsymbol{\varepsilon}_{1} \\ \vdots \\ \vdots \\ \boldsymbol{\varepsilon}_{p} \end{bmatrix} = \begin{bmatrix} \mathbf{0} \\ \vdots \\ \vdots \\ \mathbf{0} \end{bmatrix} \quad \text{and} \quad \mathbf{V}\begin{bmatrix} \boldsymbol{\varepsilon}_{i} \\ \vdots \\ \vdots \\ \boldsymbol{\varepsilon}_{p} \end{bmatrix} = \begin{bmatrix} \boldsymbol{\psi}_{1} \\ \ddots \\ \vdots \\ \boldsymbol{\varepsilon}_{p} \end{bmatrix}$$

$$\begin{aligned} &\operatorname{Var}((X_{i}) = \operatorname{Var}(a_{i}F + \epsilon_{i}) = a_{i}^{2}\sigma_{F}^{2} + \psi_{i}, \quad \text{for } i=1,...,p \\ &\operatorname{Cov}((X_{i}, X_{j}) = \operatorname{Cov}(a_{i}F + \epsilon_{i}, a_{j}F + \epsilon_{j}) = a_{i}a_{j}\sigma_{F}^{2}, \quad \text{for all } i \neq j \\ &\operatorname{Corr}((X_{i}, X_{j}) = \rho_{ij} = \frac{\operatorname{Cov}((X_{i}, X_{j})}{\sqrt{\operatorname{Var}(X_{i})}\sqrt{\operatorname{Var}(X_{j})}} = \frac{a_{i}a_{j}\sigma_{F}^{2}}{\sqrt{a_{i}^{2}\sigma_{F}^{2} + \psi_{i}}\sqrt{a_{j}^{2}\sigma_{F}^{2} + \psi_{j}}} \\ &= a_{i}a_{j} \quad \text{if } \quad \sigma_{F}^{2} = 1. \end{aligned}$$

Then the ratio of correlations is

$$\frac{\rho_{ik}}{\rho_{jk}} = \frac{\frac{a_{i}a_{k}\sigma_{F}^{2}}{\sqrt{a_{i}^{2}\sigma_{F}^{2} + \psi_{i}}\sqrt{a_{k}^{2}\sigma_{F}^{2} + \psi_{k}}}{\frac{a_{j}a_{k}\sigma_{F}^{2}}{\sqrt{a_{j}^{2}\sigma_{F}^{2} + \psi_{j}}\sqrt{a_{k}^{2}\sigma_{F}^{2} + \psi_{k}}}} = \frac{a_{i}\sqrt{a_{i}^{2}\sigma_{F}^{2} + \psi_{i}}}{a_{j}\sqrt{a_{j}^{2}\sigma_{F}^{2} + \psi_{j}}}$$

remains constant as you move across k, as long as $i \neq k$ and $j \neq k$.

(b)

Since,
$$\operatorname{Var}((X_i) = \operatorname{Var}(a_iF + \varepsilon_i) = a_i^2 \sigma_F^2 + \psi_i$$
, we have $a_i^2 = \frac{\operatorname{Var}((X_i) - \psi_i)}{\sigma_F^2}$

Since the variables are standardized we have $Var((X_i) = 1)$, and taking $\sigma_F^2 = 1$ yields $a_i^2 = 1 - \psi_i$, for i = 1, ..., p.

and

(c)

Compute $\bar{\mathbf{r}}_{k} = \frac{1}{p-1} \sum_{i \neq k} \mathbf{r}_{ik}$, for k = 1, ..., p, and $\bar{\mathbf{r}} = \frac{2}{p(p-1)} \sum_{i < j} \mathbf{r}_{ij}$

$$\hat{\gamma} = \frac{(p-1)^2(1-(1-\bar{r})^2)}{p-(p-2)(1-\bar{r})^2} \text{ and } T = \frac{n-1}{(1-\bar{r})^2} \left[\sum_{i < j} (r_{ij} - \bar{r})^2 - \hat{\gamma} \sum_{k=1}^p (\bar{r}_k - \bar{r})^2 \right] = 16.7$$

with (p+1)(p-2)/2 = 14 d.f. and p-value = .272. The hypothesis of equal correlations is not rejected.

In this case, n = 33, p = 6, $\overline{r_1} = 0.72$, $\overline{r_1} = 0.72$, $\overline{r_2} = 0.68$, $\overline{r_3} = 0.63$, $\overline{r_4} = 0.54$, $\overline{r_5} = 0.54$, $\overline{r_6} = 0.52$, $\overline{r} = 0.61$ and $\hat{\gamma} = 3.93$.

- (d) (i) A one factor model appears to be adequate. In this case, the value of the chi-squared test statistic is 2.516 on 9 d.f., with p-value = 0.989.
 - (ii) This is consistent with the result in part (c). As shown in problem 3, a one factor model is adequate when all correlations are equal.

5. pro	(a) Fact blem	Factor 1: Reading and verbal ability factor with some emphasis on			
		reasoning. Perhaps subjects with better verbal skills are better able to understand problems and express ideas for solving problems.			
	Factor 2:	Perceptual speed / mathematical ability factor			
	Factor 3:	Spatial relation and visual perception ability with some emphasis on deductive reasoning and series completion.			
	Factor 4:	Recognition ability. Subjects with higher values of this factor are better able to recognize words, numbers, figures and object- number associations.			
	Factor 5:	Subjects with higher values for this factor have greater ability to memorize figure-word associations.			

(b) The five factors produced by applying a varimax rotation to the results from iterated principal factor estimation are similar to those obtained from rotating principal component in part (a). There are some small differences.

- Factor 1: Reading and verbal ability factor.
- Factor 2: Spatial relation and visual perception ability with some emphasis on deductive reasoning and problem solving.
- Factor 3: Perceptual speed / mathematical ability factor.
- Factor 4: Word and symbol recognition and ability to memorize object-number and number-figure associations.
- Factor 5: Ability to memorize figure-word associations and perceptual speed.
- (c)Varimax rotation of five factors obtained from maximum likelihood estimation produces factors similar to those reported in part (b), but these factors may be a little easier to interpret. The third factor, for example, is more clearly a perceptual speed and mathematical ability factor. The fourth factor now combines word, number, and figure recognition with the ability to memorize object-number, number-figure, and figure-word associations.
- (d) Both the SCREE plot and the chi-square tests based on maximum likelihood estimation indicate that 5 factors are adequate and 4 factors provide a solution that is almost as good. Results for the chi-square tests are:

H_o: Five factors are sufficient. $X^2 = 186.8$ on 166 d.f. with p-value = 0.128

H_o: Four factors are sufficient. $X^2 = 226.7$ on 186 d.f. with p-value = 0.022

The fifth rotated factor explains only about 2% of the variation in the standardized scores, and many people found it difficult to interpret. The AIC criterion decreases only slightly when a fifth factor is included, while the Schwarz criterion increases. It seems reasonable to use four factors, but five factor solutions are considered below because the varimax rotation of the four factor solution essentially yields the first four factors obtained from rotating the five factor solution.

- (e) The PROMAX rotation produces factors similar to those obtained from the VARIMAX rotation, but they are more sharply focused on separate subsets of variables. These factors have moderate positive correlations of about 0.4 with each other.
- (f) Unlike the varimax and promax rotations (which makes use of a varimax rotation in an intermediate step). The quartimax rotation does not destroy the overall intelligence factor. Then it proceeds to create additional factors where difference subsets of variables have loadings on only one of the additional factors.

	Factor 1:	Overall intelligence or general performance factor.
	Factor 2:	Verbal ability and comprehension factor.
	Factor 3:	Perceptual speed / mathematical ability factor
	Factor 4:	Symbol recognition and ability to memorize object-number and number-figure associations
	Factor 5:	Object-number and figure-word association and geometry skills.
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6. (a)

$G^2 = 292.2$	d.f=126	p-value<0.001			
$X^2 = 280.697$	d.f =126	p-value<0.001			
The independen	ce model is	not appropriate.	There are significant associations		
between size categories and supplier attributes.					

(b) The users of the Beta shipper and the larger users of the alpha shipper are mostly concerned about low cost. The smaller users of the alpha shipper have a relatively strong concern about service and convenience such as easy to use, low paper work and ship anywhere. The users of the Gamma and Delta shippers are more concerned about reliability (packages are delivered to the right place on time.) Delta users and smaller users of Gamma have additional concerns about special services like the use of technology to track shipments, concern about speedy handling of claims and the ability of the shipper to make special trips. (These attributes are far away from the concern about low cost). The users of Gamma and Delta seem willing to pay more for dependable and timely deliveries and special services.