√ 5.1 a) $\bar{\mathbf{x}} = \begin{bmatrix} 6 \\ 10 \end{bmatrix}$; $S = \begin{bmatrix} 8 & -10/3 \\ -10/3 & 2 \end{bmatrix}$

$T^2 = 150/11 = 13.64$

b) T^2 is $3F_{2,2}$ (see (5-5))

c) $H_0: \boldsymbol{\mu}' = [7, 11]$

$\alpha = .05$ so $F_{2,2}(.05) = 19.00$

Since $T^2 = 13.64 < 3F_{2,2}(.05) = 3(19) = 57$; do not reject H_0 at the $\alpha = .05$ level

√ 5.3 a)
$$T^2 = \frac{(n-1)\left|\sum_{j=1}^{n}(\mathbf{x}_j-\boldsymbol{\mu}_0)(\mathbf{x}_j-\boldsymbol{\mu}_0)'\right|}{\left|\sum_{j=1}^{n}(\mathbf{x}_j-\bar{\mathbf{x}})(\mathbf{x}_j-\bar{\mathbf{x}})'\right|} - (n-1) = \frac{3(244)}{44} - 3 = 13.64$$

b)
$$\Lambda = \left(\frac{\left|\sum_{j=1}^{n}(\mathbf{x}_j-\bar{\mathbf{x}})(\mathbf{x}_j-\bar{\mathbf{x}})'\right|}{\left|\sum_{j=1}^{n}(\mathbf{x}_j-\boldsymbol{\mu}_0)(\mathbf{x}_j-\boldsymbol{\mu}_0)'\right|}\right)^{n/2} = \left(\frac{44}{244}\right)^2 = .0325$$

Wilks' lambda $= \Lambda^{2/n} = \Lambda^{1/2} = \sqrt{.0325} = .1803$

√ 5.5 $H_0: \boldsymbol{\mu}' = [.55, .60]$; $T^2 = 1.17$

$\alpha = .05$; $F_{2,40}(.05) = 3.23$

Since $T^2 = 1.17 < \frac{2(41)}{40} F_{2,40}(.05) = 2.05(3.23) = 6.62$,

we do not reject H_0 at the $\alpha = .05$ level. The result is consistent with the 95% confidence ellipse for $\boldsymbol{\mu}$ pictured in Figure 5.1 since $\boldsymbol{\mu}' = [.55, .60]$ is <u>inside</u> the ellipse.

5.7

$$\overline{x} = \begin{bmatrix} 4.640 \\ 45.400 \\ 9.965 \end{bmatrix}, S = \begin{bmatrix} 2.879 & 10.010 & -1.810 \\ 10.010 & 199.788 & -5.640 \\ -1.810 & -5.640 & 3.628 \end{bmatrix}$$

(i)

95% simultaneous T^2 confidence intervals for μ_1, μ_2, μ_3:

$$\frac{(n-1)p}{n-p}F_{p,n-p}(\alpha) = \frac{(20-1)3}{20-3}F_{3,17}(0.05) = \frac{(19)3}{17}(3.20) = 10.73$$

$$4.640 - \sqrt{10.73}\sqrt{\frac{2.879}{20}} \le \mu_1 \le 4.640 + \sqrt{10.73}\sqrt{\frac{2.879}{20}} \Longrightarrow 3.40 \le \mu_1 \le 5.88$$

$$45.400 - \sqrt{10.73}\sqrt{\frac{199.788}{20}} \le \mu_2 \le 45.400 + \sqrt{10.73}\sqrt{\frac{199.788}{20}} \Longrightarrow 35.05 \le \mu_2 \le 55.75$$

$$9.965 - \sqrt{10.73}\sqrt{\frac{3.628}{20}} \le \mu_3 \le 9.965 + \sqrt{10.73}\sqrt{\frac{3.628}{20}} \Longrightarrow 8.57 \le \mu_3 \le 11.36$$

(ii)

95% simultaneous Bonferroni confidence intervals for μ_1, μ_2, μ_3:

$$t_{n-1}\left(\frac{\alpha}{2p}\right) = t_{20-1}\left(\frac{0.05}{2 \times 3}\right) = t_{19}(0.0083) = 2.625$$

$4.640 - 2.625\sqrt{\frac{2.879}{20}} \le \mu_1 \le 4.640 + 2.625\sqrt{\frac{2.879}{20}}$ $\Longrightarrow$ ~~4.03~~ 3.64 $\le \mu_1 \le$ ~~5.25~~ 5.64

$45.400 - 2.625\sqrt{\frac{199.788}{20}} \le \mu_2 \le 45.400 + 2.625\sqrt{\frac{199.788}{20}}$ $\Longrightarrow$ ~~40.28~~ 37.10 $\le \mu_2 \le$ ~~50.52~~ 53.70

$9.965 - 2.625\sqrt{\frac{3.628}{20}} \le \mu_3 \le 9.965 + 2.625\sqrt{\frac{3.628}{20}}$ $\Longrightarrow$ ~~9.27~~ 8.85 $\le \mu_3 \le$ ~~10.66~~ 11.08

(iii)

Compare the intervals in part(i) and part(ii), we see that Bonferroni intervals provide more precise estimates than the T^2-intervals.

5.12 Initial estimates are

$$\tilde{\mu} = \begin{bmatrix} 4 \\ 6 \\ 2 \end{bmatrix}, \quad \tilde{\Sigma} = \begin{bmatrix} 0.5 & 0.0 & 0.5 \\ & 2.0 & 0.0 \\ & & 1.5 \end{bmatrix}.$$

The first revised estimates are

$$\tilde{\mu} = \begin{bmatrix} 4.0833 \\ 6.0000 \\ 2.2500 \end{bmatrix}, \quad \tilde{\Sigma} = \begin{bmatrix} 0.6042 & 0.1667 & 0.8125 \\ & 2.500 & 0.0 \\ & & 1.9375 \end{bmatrix}.$$

5.13 The χ^2 distribution with 3 degrees of freedom.

5.14 Length of one-at-a time t-interval / Length of Bonferroni interval $= t_{n-1}(\alpha/2)/t_{n-1}(\alpha/2m)$.

	m		
n	2	4	10
15	0.8546	0.7489	0.6449
25	0.8632	0.7644	0.6678
50	0.8691	0.7749	0.6836
100	0.8718	0.7799	0.6910
∞	0.8745	0.7847	0.6983

5.15

(a).

$$E(X_{ij}) = (1)p_i + (0)(1-p_i) = p_i.$$
$$Var(X_{ij}) = (1-p_i)^2 p_i + (0-p_i)^2(1-p_i) = p_i(1-p_i)$$

(b). $Cov(X_{ij}, X_{kj}) = E(X_{ij}X_{ik}) - E(X_{ij})E(X_{kj}) = 0 - p_i p_k = -p_i p_k.$

5.16

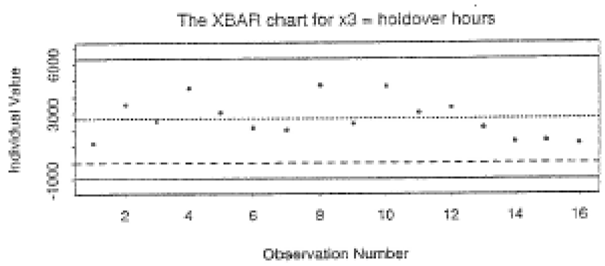
(a). Using $\hat{p}_j \pm \sqrt{\chi_4^2(0.05)}\sqrt{\hat{p}_j(1-\hat{p}_j)/n}$, the 95 % confidence intervals for p_1, p_2, p_3, p_4, p_5 are (0.221, 0.370), (0.258, 0.412), (0.098, 0.217), (0.029, 0.112), (0.084, 0.198) respectively.

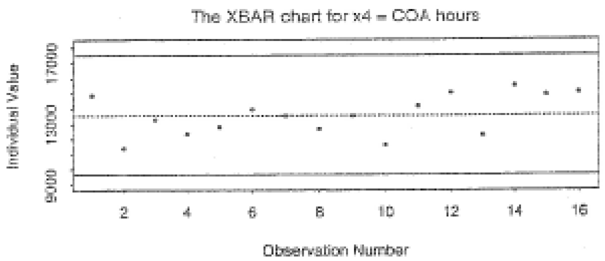
(b). Using $\hat{p}_1 - \hat{p}_2 \pm \sqrt{\chi_4^2(0.05)}\sqrt{(\hat{p}_1(1-\hat{p}_1) + \hat{p}_2(1-\hat{p}_2) - 2\hat{p}_1\hat{p}_2)/n}$, the 95 % confidence interval for $p_1 - p_2$ is (−0.118, 0.0394). There is no significant difference in two proportions.

Or bank B slightly larger.

5.24 Individual $\overline{X}$ charts for the Madison, Wisconsin, Police Department data

	xbar	s	LCL	UCL	
LegalOT	3557.8	606.5	1738.1	5377.4	
ExtraOT	1478.4	1182.8	-2070.0	5026.9	use LCL = 0
Holdover	2676.9	1207.7	-946.2	6300.0	use LCL = 0
COA	13563.6	1303.2	9654.0	17473.2	
MeetOT	800.0	474.0	-622.1	2222.1	use LCL = 0

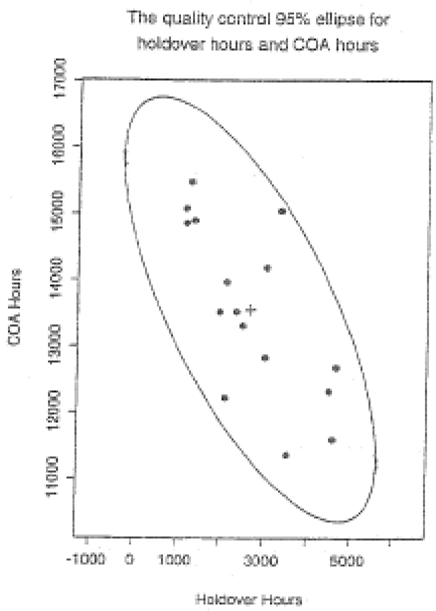


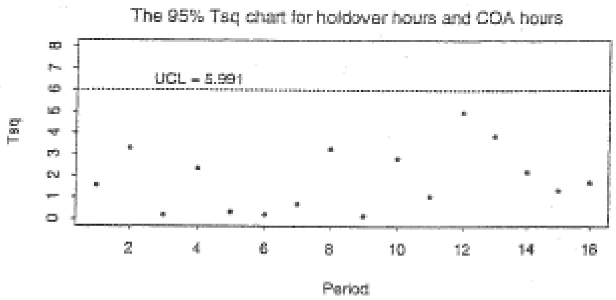


Both holdover and COA hours are stable and in control.

5.25 Quality ellipse and T^2 chart for the holdover and COA overtime hours. All points are in control. The quality control 95% ellipse is

$$1.37\times 10^{-6}(x_3 - 2677)^2 + 1.18\times 10^{-6}(x_4 - 13564)^2$$
$$+1.80\times 10^{-6}(x_3 - 2677)(x_4 - 13564) = 5.99.$$





5.27 The 95% prediction ellipse for x_3 = holdover hours and x_4 = COA hours is

$$1.37\times 10^{-6}(x_3 - 2677)^2 + 1.18\times 10^{-6}(x_4 - 13564)^2$$
$$+1.80\times 10^{-6}(x_3 - 2677)(x_4 - 13564) = 8.51.$$