√ 6.6 a) Treatment 2: Sample mean vector $\begin{bmatrix} 2 \\ 4 \end{bmatrix}$; sample covariance matrix

$$\begin{bmatrix} 1 & -3/2 \\ -3/2 & 3 \end{bmatrix}$$

Treatment 3: Sample mean vector $\begin{bmatrix} 3 \\ 2 \end{bmatrix}$; sample covariance matrix

$$\begin{bmatrix} 2 & -4/3 \\ -4/3 & 4/3 \end{bmatrix}$$

$$S_{pooled} = \begin{bmatrix} 1.6 & \\ -1.4 & 2 \end{bmatrix}$$

b) $$T^2 = [2-3,\ 4-2] \left[\left(\frac{1}{3} + \frac{1}{4}\right) \begin{bmatrix} 1.6 & -1.4 \\ -1.4 & 2 \end{bmatrix} \right]^{-1} \begin{bmatrix} 2-3 \\ 4-2 \end{bmatrix} = 3.88$$

$$\frac{(n_1+n_2-2)p}{(n_1+n_2-p-1)} F_{p,n_1+n_2-p-1}(.01) = \frac{(5)2}{4}(18) = 45$$

Since $T^2 = 3.88 < 45$ do not reject $H_0: \underline{\mu}_2 - \underline{\mu}_3 = \underline{0}$ at the $\alpha = .01$ level.

c) 99% simultaneous confidence intervals:

$$\mu_{21} - \mu_{31}: (2-3) \pm \sqrt{45}\ \sqrt{\left(\frac{1}{3}+\frac{1}{4}\right)1.6} = -1 \pm 6.5$$

$$\mu_{22} - \mu_{32}: 2 \pm 7.2$$

6.8 a) For first variable:

observation = mean + treatment effect + residual

$$\begin{bmatrix} 6 & 5 & 8 & 4 & 7 \\ 3 & 1 & 2 & & \\ 2 & 5 & 3 & 2 & \end{bmatrix} = \begin{bmatrix} 4 & 4 & 4 & 4 & 4 \\ 4 & 4 & 4 & & \\ 4 & 4 & 4 & 4 & \end{bmatrix} + \begin{bmatrix} 2 & 2 & 2 & 2 & 2 \\ -2 & -2 & -2 & & \\ -1 & -1 & -1 & -1 & \end{bmatrix} + \begin{bmatrix} 0 & -1 & 2 & -2 & 1 \\ 1 & -1 & 0 & & \\ -1 & 2 & 0 & -1 & \end{bmatrix}$$

$SS_{obs} = 246$ $\qquad SS_{mean} = 192$ $\qquad SS_{tr} = 36$ $\qquad SS_{res} = 18$

For second variable:

$$\begin{bmatrix} 7 & 9 & 6 & 9 & 9 \\ 3 & 6 & 3 & & \\ 3 & 1 & 1 & 3 & \end{bmatrix} = \begin{bmatrix} 5 & 5 & 5 & 5 & 5 \\ 5 & 5 & 5 & & \\ 5 & 5 & 5 & 5 & \end{bmatrix} + \begin{bmatrix} 3 & 3 & 3 & 3 & 3 \\ -1 & -1 & -1 & & \\ -3 & -3 & -3 & -3 & \end{bmatrix} + \begin{bmatrix} -1 & 1 & -2 & 1 & 1 \\ -1 & 2 & -1 & & \\ 1 & -1 & -1 & 1 & \end{bmatrix}$$

$SS_{obs} = 402$ $\qquad SS_{mean} = 300$ $\qquad SS_{tr} = 84$ $\qquad SS_{res} = 18$

Cross product contributions:

275 240 48 -13

b) MANOVA table:

Source of Variation	SSP	d.f.
Treatment	$B = \begin{bmatrix} 36 & 48 \\ 48 & 84 \end{bmatrix}$	3 - 1 = 2
Residual	$W = \begin{bmatrix} 18 & -13 \\ -13 & 18 \end{bmatrix}$	5 + 3 + 4 - 3 = 9
Total (corrected)	$\begin{bmatrix} 54 & 35 \\ 35 & 102 \end{bmatrix}$	11

c) $\Lambda^* = \frac{|W|}{|B+W|} = \frac{155}{4283} = .0362$

Using Table 6.3 with $p = 2$ and $g = 3$

$$\left(\frac{1 - \sqrt{\Lambda^*}}{\sqrt{\Lambda^*}}\right)\left(\frac{\Sigma n_\ell - g - 1}{g - 1}\right) = 17.02 .$$

Since $F_{4,16}(.01) = 4.77$ we conclude that treatment differences exist at $\alpha = .01$ level.

Alternatively, using Bartlett's procedure,

$$-\left(n - 1 - \frac{(p+g)}{2}\right) \ln \Lambda^* = -(12 - 1 - \tfrac{5}{2})\ln(.0362) = 28.209$$

Since $\chi_4^2(.01) = 13.28$ we again conclude treatment differences exist at $\alpha = .01$ level.

6.11 $L(\underset{\sim}{\mu}_1, \underset{\sim}{\mu}_2, \Sigma) = L(\underset{\sim}{\mu}_1, \Sigma) L(\underset{\sim}{\mu}_2, \Sigma)$

$$= \left[\frac{1}{(2\pi)^{\frac{(n_1+n_2)p}{2}} |\Sigma|^{\frac{n_1+n_2}{2}}}\right] \exp\left\{-\frac{1}{2}\left(\text{tr } \Sigma^{-1}[(n_1-1)S_1 + (n_2-1)S_2]\right.\right.$$

$$\left.\left. + n_1(\bar{\underset{\sim}{x}}_1 - \underset{\sim}{\mu}_1)' \Sigma^{-1}(\bar{\underset{\sim}{x}}_1 - \underset{\sim}{\mu}_1) + n_2(\bar{\underset{\sim}{x}}_2 - \underset{\sim}{\mu}_2)' \Sigma^{-1}(\bar{\underset{\sim}{x}}_2 - \underset{\sim}{\mu}_2)\right)\right\}$$

using (4-16) and (4-17). The likelihood is maximized with respect to $\underset{\sim}{\mu}_1$ and $\underset{\sim}{\mu}_2$ at $\hat{\underset{\sim}{\mu}}_1 = \bar{\underset{\sim}{x}}_1$ and $\hat{\underset{\sim}{\mu}}_2 = \bar{\underset{\sim}{x}}_2$ respectively and with respect to Σ at

$$\hat{\Sigma} = \frac{1}{n_1+n_2}[(n_1 - 1)S_1 + (n_2 - 2)S_2] = \left(\frac{n_1+n_2-2}{n_1+n_2}\right) S_{pooled}$$

(For the maximization with respect to Σ see Result 4.10 with $b = \frac{n_1+n_2}{2}$ and $B = (n_1 - 1)S_1 + (n_2 - 2)S_2$)

6.13 a) and b) For first variable:

Observation = mean + factor 1 effect + factor 2 effect + residual

$$\begin{bmatrix} 6 & 4 & 8 & 2 \\ 3 & -3 & 4 & -4 \\ -3 & -4 & 3 & -4 \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \end{bmatrix} + \begin{bmatrix} 4 & 4 & 4 & 4 \\ -1 & -1 & -1 & -1 \\ -3 & -3 & -3 & -3 \end{bmatrix} + \begin{bmatrix} 1 & -2 & 4 & -3 \\ 1 & -2 & 4 & -3 \\ 1 & -2 & 4 & -3 \end{bmatrix} + \begin{bmatrix} 0 & 1 & -1 & 0 \\ 2 & -1 & 0 & -1 \\ -2 & 0 & 1 & 1 \end{bmatrix}$$

$SS_{tot} = 220$ $SS_{mean} = 12$ $SS_{fac\,1} = 104$ $SS_{fac\,2} = 90$ $SS_{res} = 14$

For second variable:

$$\begin{bmatrix} 8 & 6 & 12 & 6 \\ 8 & 2 & 3 & 3 \\ 2 & -5 & -3 & -6 \end{bmatrix} = \begin{bmatrix} 3 & 3 & 3 & 3 \\ 3 & 3 & 3 & 3 \\ 3 & 3 & 3 & 3 \end{bmatrix} + \begin{bmatrix} 5 & 5 & 5 & 5 \\ 1 & 1 & 1 & 1 \\ -6 & -6 & -6 & -6 \end{bmatrix} + \begin{bmatrix} 3 & -2 & 1 & -2 \\ 3 & -2 & 1 & -2 \\ 3 & -2 & 1 & -2 \end{bmatrix} + \begin{bmatrix} -3 & 0 & 3 & 0 \\ 1 & 0 & -2 & 1 \\ 2 & 0 & -1 & -1 \end{bmatrix}$$

$SS_{tot} = 440$ $SS_{mean} = 108$ $SS_{fac\,1} = 248$ $SS_{fac\,2} = 54$ $SS_{res} = 30$

Sum of cross products:

$$SCP_{tot} = SCP_{mean} + SCP_{fac\,1} + SCP_{fac\,2} + SCP_{res}$$

$$227 = 36 + 148 + 51 - 8$$

c) MANOVA table:

Source of Variation	SSP	d.f.
Factor 1	$\begin{bmatrix} 104 & 148 \\ 148 & 248 \end{bmatrix}$	$g-1 = 3-1 = 2$
Factor 2	$\begin{bmatrix} 90 & 51 \\ 51 & 54 \end{bmatrix}$	$b-1 = 4-1 = 3$
Residual	$\begin{bmatrix} 14 & -8 \\ -8 & 30 \end{bmatrix}$	$(g-1)(b-1) = 6$
Total (Corrected)	$\begin{bmatrix} 208 & 191 \\ 191 & 332 \end{bmatrix}$	$gb-1 = 11$

d) We reject $H_0: \underset{\sim}{\tau}_1 = \underset{\sim}{\tau}_2 = \underset{\sim}{\tau}_3 = \underset{\sim}{0}$ at $\alpha = .05$ level since

$$-\left[(g-1)(b-1) - \left(\frac{p+1-(g-1)}{2}\right)\right]\ln\Lambda^* = -\left[6 - \frac{3-2}{2}\right]\ln\left(\frac{|SS_{res}|}{|SSP_{fac\,1} + SSP_{res}|}\right)$$

$$= -5.5\ln\left(\frac{356}{13204}\right) = 19.87 > \chi^2_4(.05) = 9.49$$

and conclude there are factor 1 effects.

We also reject $H_0: \underset{\sim}{\beta}_1 = \underset{\sim}{\beta}_2 = \underset{\sim}{\beta}_3 = \underset{\sim}{\beta}_4 = \underset{\sim}{0}$ at the $\alpha = .05$ level since

$$- [(g-1)(b-1) - (\frac{p+1-(b-1)}{2})\ln\Lambda^* = -[6 - \frac{3-3}{2}]\ln\left(\frac{|SSP_{res}|}{|SSP_{fac\,2} + SSP_{res}|}\right)$$

$$= -6\ln\left(\frac{356}{6887}\right) = 17.77 > \chi^2_6(.05) = 12.59$$

and conclude there are factor 2 effects.

6.14 b) MANOVA Table:

Source of Variation	SSP	d.f.
Factor 1	$\begin{bmatrix} 496 & 184 \\ 184 & 208 \end{bmatrix}$	2
Factor 2	$\begin{bmatrix} 36 & 24 \\ 24 & 36 \end{bmatrix}$	3
Interaction	$\begin{bmatrix} 32 & 0 \\ 0 & 44 \end{bmatrix}$	6
Residual	$\begin{bmatrix} 312 & -84 \\ -84 & 400 \end{bmatrix}$	12
Total (Corrected)	$\begin{bmatrix} 876 & 124 \\ 124 & 688 \end{bmatrix}$	23

c) Since $-[gb(n-1) - (p+1-(g-1)(b-1))/2]\ln\Lambda^* = -13.5\ln\left(\frac{|SSP_{res}|}{|SSP_{int} + SSP_{res}|}\right)$

$= -13.5\ln(.808) = 2.88 < \chi^2_{12}(.05) = 21.03$ we <u>do not reject</u>

$H_0: \gamma_{11} = \gamma_{12} = \cdots = \gamma_{34} = 0$ (no interaction effects) at the

$\alpha = .05$ level.

Since

$$-[gb(n-1)-(p+1-(g-1))/2]\ln\Lambda^* = -11.5\ln\left(\frac{|SSP_{res}|}{|SSP_{fac\,1} + SSP_{res}|}\right)$$

$= -11.5\ln(.2447) = 16.19 > \chi_4^2(.05) = 9.49$ we reject

$H_0: \underset{\sim}{\tau}_1 = \underset{\sim}{\tau}_2 = \underset{\sim}{\tau}_3 = \underset{\sim}{0}$ (no factor 1 effects) at the $\alpha = .05$

level.

Since

$$-[gb(n-1)-(p+1-(b-1))/2]\ln\Lambda^* = -12\ln\left(\frac{|SSP_{res}|}{|SSP_{fac\,2} + SSP_{res}|}\right)$$

$= -12\ln(.7949) = 2.76 < \chi_6^2(.05) = 12.59$ we do not reject

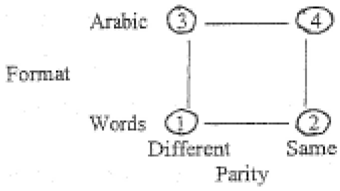
$H_0: \underset{\sim}{\beta}_1 = \underset{\sim}{\beta}_2 = \underset{\sim}{\beta}_3 = \underset{\sim}{\beta}_4 = \underset{\sim}{0}$ (no factor 2 effects) at the

$\alpha = .05$ level.

(d)

7

6.17 a)



Effects	Contrast
Parity main:	$(\mu_2+\mu_4)-(\mu_1+\mu_3)$
Format main:	$(\mu_3+\mu_4)-(\mu_1+\mu_2)$
Interaction:	$(\mu_2+\mu_3)-(\mu_1+\mu_4)$

Contrast matrix:

$$C=\begin{pmatrix}-1 & 1 & -1 & 1\\ -1 & -1 & 1 & 1\\ -1 & 1 & 1 & -1\end{pmatrix}$$

Since $T^2=153.7>\frac{31(3)}{29}(2.93)=9.40$, reject $H_0: \mathbf{C}\boldsymbol{\mu}=\mathbf{0}$ (no treatment effects) at the 5 % level.

b) 95% simultaneous T^2 intervals for the contrasts:

Parity main effect: $-206.4\pm\sqrt{9.40}\sqrt{\frac{19{,}577.5}{32}}$ → (−282.2, −130.6)

Format main effect: $-307\pm\sqrt{9.40}\sqrt{\frac{40{,}269.3}{32}}$ → (−415.8, −198.2)

Interaction effect: $22.4\pm\sqrt{9.40}\sqrt{\frac{10{,}007.3}{32}}$ → (−31.8, 76.6)

No interaction effect. Parity effect—"different" responses slower than "same" responses. Format effect—"words" slower than "Arabic".

c) The M model of numerical cognition is supported by this experiment.

d) The multivariate normal model is a reasonable population model for the scores corresponding to the parity contrast, the format contrast and the interaction contrast.

6.24 Wilks' lambda: $\Lambda^* = .8301$. Since $g = 3$, $\left(\frac{90-4-2}{4}\right)\left(\frac{1-\sqrt{.8301}}{\sqrt{.8301}}\right) = 2.049$ is an F value with 8 and 168 degrees of freedom. Since p-value $= P(F > 2.049) = .044$, we would just reject the null hypothesis $H_0 : \underset{\sim}{\tau}_1 = \underset{\sim}{\tau}_2 = \underset{\sim}{\tau}_3 = \underset{\sim}{0}$ at the 5% level implying there is a time period effect.

F statistics and p-values for ANOVA's:

	F	p-value
MaxBrth:	3.66	.030
BasHght:	0.47	.629
BasLgth:	3.84	.025
NasHght:	0.10	.901

Any differences over time periods are probably due to changes in maximum breath of skull (MaxBrth) and basialveolar length of skull (BasLgth).

95% Bonferroni simultaneous intervals: $m = pg(g-1)/2 = 12$,
$t_{87}(.05/24) = 2.94$

BasBrth

$\tau_{11} - \tau_{21}: \ -1 \pm 2.94\sqrt{\frac{1785.4}{87}\left(\frac{1}{30}+\frac{1}{30}\right)} \rightarrow -1 \pm 3.44$

$\tau_{11} - \tau_{31}: \ -3.1 \pm 3.44$

$\tau_{21} - \tau_{31}: \ -2.1 \pm 3.44$

BasHght

$\tau_{12} - \tau_{22}: \ 0.9 \pm 2.94\sqrt{\frac{1924.3}{87}\left(\frac{1}{30}+\frac{1}{30}\right)} \rightarrow 0.9 \pm 3.57$

$\tau_{12} - \tau_{32}: \ -0.2 \pm 3.57$

$\tau_{22} - \tau_{32}: \ -1.1 \pm 3.57$

BasLgth

$\tau_{13} - \tau_{23}: \ 0.10 \pm 2.94\sqrt{\frac{2153}{87}\left(\frac{1}{30}+\frac{1}{30}\right)} \rightarrow 0.10 \pm 3.78$

$\tau_{13} - \tau_{33}: \ 3.14 \pm 3.78$

$\tau_{23} - \tau_{33}: \ 3.03 \pm 3.78$

NasHgth

$\tau_{14} - \tau_{24}: \ 0.30 \pm 2.94\sqrt{\frac{840.2}{87}\left(\frac{1}{30}+\frac{1}{30}\right)} \rightarrow 0.30 \pm 2.36$

$\tau_{14} - \tau_{34}: \ -0.03 \pm 2.36$

$\tau_{24} - \tau_{34}: \ -0.33 \pm 2.36$

All the simultaneous intervals include 0. Evidence for changes in skull size over time is marginal. If changes exist, then these changes might be in maximum breath and basialveolar length of skull from time periods 1 to 3.

The usual MANOVA assumptions appear to be satisfied for these data.

6.26 To test for parallelism, consider H_{01}: $C\underset{\sim}{\mu}_1 = C\underset{\sim}{\mu}_2$ with C given by (6-61).

$$C(\bar{\underset{\sim}{x}}_1 - \bar{\underset{\sim}{x}}_2) = \begin{bmatrix} -.413 \\ -.167 \\ -.036 \end{bmatrix}; \quad (CS_{pooles}C')^{-1} = \begin{bmatrix} 1.674 & .947 & .616 \\ & 2.014 & 1.144 \\ & & 2.341 \end{bmatrix}$$

$T^2 = 9.58 > c^2 = 8.0$, we reject H_0 at the $\alpha = .05$ level. The excess electrical usage of the test group was much lower than that of the control group for the 11 A.M., 1 P.M. and 3 P.M. hours. The similar 9 A.M. usage for the two groups contradicts the parallelism hypothesis.

/6.31 (a) Two-factor MANOVA of peanuts data

E = Error SS&CP Matrix

	X1	X2	X3
X1	104.205	49.365	76.48
X2	49.365	352.105	121.995
X3	76.48	121.995	94.835

H = Type III SS&CP Matrix for FACTOR1 (Location)

	X1	X2	X3
X1	0.7008333333	-10.6575	7.1291666667
X2	-10.6575	162.0675	-108.4125
X3	7.1291666667	-108.4125	72.520833333

Manova Test Criteria and Exact F Statistics for
the Hypothesis of no Overall FACTOR1 Effect
H = Type III SS&CP Matrix for FACTOR1 E = Error SS&CP Matrix

S=1 M=0.5 N=1

Statistic	Value	F	Num DF	Den DF	Pr > F
Wilks' Lambda	0.10651620	11.1843	3	4	0.0205
Pillai's Trace	0.89348380	11.1843	3	4	0.0205
Hotelling-Lawley Trace	8.38824348	11.1843	3	4	0.0205
Roy's Greatest Root	8.38824348	11.1843	3	4	0.0205

H = Type III SS&CP Matrix for FACTOR2 (Variety)

	X1	X2	X3
X1	196.115	365.1825	42.6275
X2	365.1825	1089.015	414.655
X3	42.6275	414.655	284.10166667

Manova Test Criteria and F Approximations for
the Hypothesis of no Overall FACTOR2 Effect
H = Type III SS&CP Matrix for FACTOR2 E = Error SS&CP Matrix

S=2 M=0 N=1

Statistic	Value	F	Num DF	Den DF	Pr > F
Wilks' Lambda	0.01244417	10.6191	6	8	0.0019
Pillai's Trace	1.70910921	9.7924	6	10	0.0011
Hotelling-Lawley Trace	21.37567504	10.6878	6	6	0.0055
Roy's Greatest Root	18.18761127	30.3127	3	5	0.0012

H = Type III SS&CP Matrix for FACTOR1*FACTOR2

	X1	X2	X3
X1	205.10166667	363.6675	107.78583333
X2	363.6675	780.695	254.22
X3	107.78583333	254.22	85.951666667

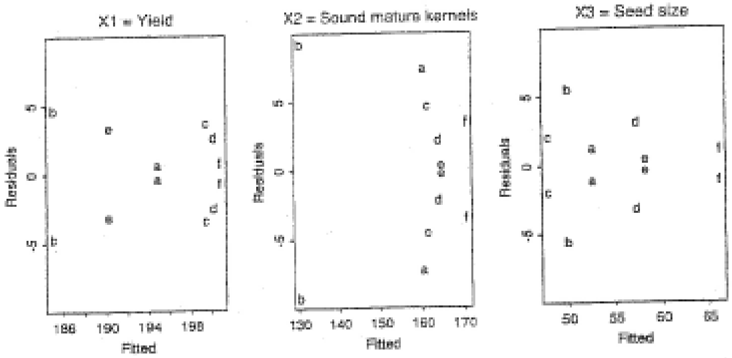
```
Manova Test Criteria and F Approximations for
the Hypothesis of no Overall FACTOR1*FACTOR2 Effect
H = Type III SS&CP Matrix for FACTOR1*FACTOR2   E = Error SS&CP Matrix

S=2    M=0    N=1
Statistic                      Value          F      Num DF    Den DF   Pr > F
Wilks' Lambda             0.07429984     3.5582           6         8   0.0508
Pillai's Trace            1.29086073     3.0339           6        10   0.0587
Hotelling-Lawley Trace    7.54429038     3.7721           6         6   0.0655
Roy's Greatest Root       6.82409388    11.3735           3         5   0.0113
```

(b) Residual analysis. The residuals for X_2 at location 2 and variety 5 look large.

CODE	FACTOR1	FACTOR2	PRED1	RES1	PRED2	RES2	PRED3	RES3
a	1	5	194.80	0.50	160.40	-7.30	52.55	-1.15
a	1	5	194.80	-0.50	160.40	7.30	52.55	1.15
b	2	5	185.05	4.65	130.30	9.20	49.95	5.55
b	2	5	185.05	-4.65	130.30	-9.20	49.95	-5.55
c	1	6	199.45	3.55	161.40	-4.60	47.80	2.00
c	1	6	199.45	-3.55	161.40	4.60	47.80	-2.00
d	2	6	200.15	2.55	163.95	2.15	57.25	3.15
d	2	6	200.15	-2.55	163.95	-2.15	57.25	-3.15
e	1	8	190.25	3.25	164.80	-0.30	58.20	-0.40
e	1	8	190.25	-3.25	164.80	0.30	58.20	0.40
f	2	8	200.75	0.75	170.30	-3.50	66.10	-1.10
f	2	8	200.75	-0.75	170.30	3.50	66.10	1.10

Plot of residuals versus fitted values



(c) Univariate two-factor ANOVA. With $\alpha = 0.05$, from Wilk's lambda test in part (a), the interaction term can be dropped.

```
Dependent Variable: X1
                 R-Square           C.V.       Root MSE                X1 Mean
                 0.388870       3.187484        6.21798                195.075

Source                 DF    Type III SS    Mean Square   F Value     Pr > F
FACTOR1                 1       0.700833       0.700833      0.02     0.8962
FACTOR2                 2     196.115000      98.057500      2.54     0.1403

Dependent Variable: X2
                 R-Square           C.V.       Root MSE                X2 Mean
                 0.524809       7.506437        11.8996                158.525

Source                 DF    Type III SS    Mean Square   F Value     Pr > F
FACTOR1                 1      162.06750      162.06750      1.14     0.3159
FACTOR2                 2     1089.01500      544.50750      3.85     0.0676

Dependent Variable: X3
                 R-Square           C.V.       Root MSE                X3 Mean
                 0.663596       8.595035        4.75377                55.3083

Source                 DF    Type III SS    Mean Square   F Value     Pr > F
FACTOR1                 1      72.520833      72.520833      3.21     0.1110
FACTOR2                 2     284.101667     142.050833      6.29     0.0229
```

(d) Bonferroni simultaneous comparisons of variety.
Only varieties 5 and 8 differ, and they differ only on X_3.

```
Bonferroni (Dunn) T tests for variable: X1
Alpha= 0.05  Confidence= 0.95  df= 8  MSE= 38.66333
Critical Value of T= 3.01576
Minimum Significant Difference= 13.26
Comparisons significant at the 0.05 level are indicated by '***'.

                    Simultaneous              Simultaneous
                           Lower   Difference        Upper
   FACTOR2            Confidence      Between   Confidence
  Comparison               Limit        Means        Limit
6     - 8                 -8.960        4.300       17.560
6     - 5                 -3.385        9.875       23.135
8     - 6                -17.560       -4.300        8.960
8     - 5                 -7.685        5.575       18.835
5     - 6                -23.135       -9.875        3.385
5     - 8                -18.835       -5.575        7.685
```

```
Bonferroni (Dunn) T tests for variable: X2
Alpha= 0.05  Confidence= 0.95  df= 8  MSE= 141.6
Critical Value of T= 3.01576
Minimum Significant Difference= 25.375
Comparisons significant at the 0.05 level are indicated by '***'.
```

FACTOR2 Comparison	Simultaneous Lower Confidence Limit	Difference Between Means	Simultaneous Upper Confidence Limit	
8 - 6	-20.500	4.875	30.250	
8 - 5	-3.175	22.200	47.575	
6 - 8	-30.250	-4.875	20.500	
6 - 5	-8.050	17.325	42.700	
5 - 8	-47.575	-22.200	3.175	
5 - 6	-42.700	-17.325	8.050	

```
Bonferroni (Dunn) T tests for variable: X3
Alpha= 0.05  Confidence= 0.95  df= 8  MSE= 22.59833
Critical Value of T= 3.01576
Minimum Significant Difference= 10.137
Comparisons significant at the 0.05 level are indicated by '***'.
```

FACTOR2 Comparison	Simultaneous Lower Confidence Limit	Difference Between Means	Simultaneous Upper Confidence Limit	
8 - 6	-0.512	9.625	19.762	
8 - 5	0.763	10.900	21.037	***
6 - 8	-19.762	-9.625	0.512	
6 - 5	-8.862	1.275	11.412	
5 - 8	-21.037	-10.900	-0.763	***
5 - 6	-11.412	-1.275	8.862	

6.35 Fitting a quadratic growth curve to calcium measurements on the dominant ulna, treating all 31 subjects as a single group.

```
XBAR                 MLE of beta          [B'Sp^(-1)B]^(-1)
70.7839              71.6039              92.2789 -5.9783  0.0799
71.9323               3.8673              -5.9783  9.3020 -2.9033
71.8065              -1.9404               0.0799 -2.9033  1.0760
64.6548

S                                         W = (n-1)*S
94.5441 90.7962 80.0081 78.0676           2836.322 2723.886 2400.243 2342.027
90.7962 93.6616 78.9965 77.7725           2723.886 2809.848 2369.894 2333.175
80.0081 78.9965 77.1546 70.0366           2400.243 2369.894 2314.639 2101.099
78.0676 77.7725 70.0366 75.9319           2342.027 2333.175 2101.099 2277.957

Estimated covariance matrix               W2
 3.1894 -0.2066  0.0028                   2857.167 2764.522 2394.410 2369.674
-0.2066  0.3215 -0.1003                   2764.522 2889.063 2358.522 2387.070
 0.0028 -0.1003  0.0372                   2394.410 2358.522 2316.271 2093.362
                                          2369.674 2387.070 2093.362 2314.625

Lambda = |W|/|W2| = 0.7653
```

Since, with $\alpha = 0.01$, $-\left[n - \frac{1}{2}(p - q + 1)\right]\log(\Lambda) = 7.893 > \chi^2_{4-2-1}(0.01) = 6.635$, we reject the null hypothesis of a quadratic fit at $\alpha = 0.01$.