7.1 $$\hat{\boldsymbol\beta} = (Z'Z)^{-1}Z'\mathbf{y} = \frac{1}{120}\begin{bmatrix}120 & -10\\ -10 & 1\end{bmatrix}\begin{bmatrix}72\\ 872\end{bmatrix} = \frac{1}{15}\begin{bmatrix}-10\\ 19\end{bmatrix} = \begin{bmatrix}-.667\\ 1.267\end{bmatrix}$$

$$\hat{\mathbf{y}} = Z\hat{\boldsymbol\beta} = \frac{1}{15}\begin{bmatrix}180\\ 85\\ 123\\ 351\\ 199\\ 142\end{bmatrix} = \begin{bmatrix}12.000\\ 5.667\\ 8.200\\ 23.400\\ 13.267\\ 9.467\end{bmatrix}\ ; \quad \hat{\boldsymbol\varepsilon} = \mathbf{y} - \hat{\mathbf{y}} = \begin{bmatrix}15\\ 9\\ 3\\ 25\\ 9\\ 13\end{bmatrix} - \begin{bmatrix}12.000\\ 5.667\\ 8.200\\ 23.400\\ 13.267\\ 9.467\end{bmatrix} = \begin{bmatrix}3.000\\ 3.333\\ -5.200\\ 1.600\\ -6.267\\ 3.533\end{bmatrix}$$

Residual sum of squares: $\hat{\boldsymbol\varepsilon}'\hat{\boldsymbol\varepsilon} = 101.467$

Fitted equation: $\hat{y} = -.667 + 1.267\, z_1$

7.3 Follow hint and note that $\hat{\boldsymbol\varepsilon}^* = \mathbf{Y}^* - \hat{\mathbf{Y}}^* = V^{-1/2}\mathbf{Y} - V^{-1/2}Z\boldsymbol\beta_W$ and $(n-r-1)\sigma^2 = \hat{\boldsymbol\varepsilon}^{*\prime}\hat{\boldsymbol\varepsilon}^*$ is distributed as χ^2_{n-r-1}.

7.4 a) $V = I$ so $\hat{\boldsymbol\beta}_W = (\mathbf{z}'\mathbf{z})^{-1}\mathbf{z}'\mathbf{y} = (\sum_{j=1}^{n} z_j y_j)/(\sum_{j=1}^{n} z_j^2)$.

b) V^{-1} is diagonal with $j^{\underline{th}}$ diagonal element $1/z_j$ so

$$\hat{\boldsymbol\beta}_W = (\mathbf{z}'V^{-1}\mathbf{z})^{-1}\,\mathbf{z}'V^{-1}\mathbf{y} = (\sum_{j=1}^{n} y_j)/(\sum_{j=1}^{n} z_j)$$

c) V^{-1} is diagonal with $j^{\underline{th}}$ diagonal element $1/z_j^2$ so

$$\hat{\boldsymbol\beta}_W = (\mathbf{z}'V^{-1}\mathbf{z})^{-1}\mathbf{z}'V^{-1}\mathbf{y} = (\sum_{j=1}^{n}(y_j/z_j))/n$$

7.6 a) First note that $\Lambda^- = \text{diag}[\lambda_1^{-1},\dots,\lambda_{r_1+1}^{-1},0,\dots 0]$ is a generalized inverse of Λ since

$$\Lambda\Lambda^- = \begin{bmatrix} I_{r_1+1} & 0 \\ 0 & 0 \end{bmatrix} \quad \text{so} \quad \Lambda\Lambda^-\Lambda = \begin{bmatrix} \lambda_1 & & & & 0 \\ & \ddots & & & \\ & & \lambda_{r_1+1} & & \\ & & & 0 & \\ 0 & & & & \ddots 0 \end{bmatrix} = \Lambda$$

Since
$$Z'Z = \sum_{i=1}^{p} \lambda_i \mathbf{e}_i \mathbf{e}_i' = P\Lambda P'$$

$$(Z'Z)^- = \sum_{i=1}^{r_1+1} \lambda_i^{-1} \mathbf{e}_i \mathbf{e}_i' = P\Lambda^- P'$$

with $PP' = P'P = I_p$, we check that the defining relation holds

$$\begin{aligned}(Z'Z)(Z'Z)^-(Z'Z) &= P\Lambda \underbrace{P'(P}\Lambda^- \underbrace{P')P}\Lambda P' \\ &= P\Lambda\Lambda^-\Lambda P' \\ &= P\Lambda P' = Z'Z\end{aligned}$$

b) By the hint, if $Z\hat{\boldsymbol{\beta}}$ is the projection, $\mathbf{0} = Z'(\mathbf{y} - Z\hat{\boldsymbol{\beta}})$ or $Z'Z\hat{\boldsymbol{\beta}} = Z'\mathbf{y}$. In c), we show that $Z\hat{\boldsymbol{\beta}}$ is the projection of $\mathbf{y}$.

c) Consider $\mathbf{q}_i = \lambda_i^{-1/2} Z\mathbf{e}_i$ for $i = 1,2,\dots,r_1+1$. Then

$$Z(Z'Z)^-Z' = Z\left(\sum_{i=1}^{r_1+1} \lambda_i^{-1} \mathbf{e}_i\mathbf{e}_i'\right)Z' = \sum_{i=1}^{r_1+1} \mathbf{q}_i\mathbf{q}_i'$$

The $\{\mathbf{q}_i\}$ are r_1+1 mutually perpendicular unit length vectors that span the space of all linear combinations of the columns of Z. The projection of $\mathbf{y}$ is then (see Result 2A.2 and Definition 2A.12)

$$\sum_{i=1}^{r_1+1} (\mathbf{q}_i'\mathbf{y})\mathbf{q}_i = \sum_{i=1}^{r_1+1} \mathbf{q}_i(\mathbf{q}_i'\mathbf{y}) = \left(\sum_{i=1}^{r_1+1} \mathbf{q}_i\mathbf{q}_i'\right)\mathbf{y} = Z(Z'Z)^- Z'\mathbf{y}$$

d) See Hint.

7.9

$$Z' = \begin{bmatrix} 1 & 1 & 1 & 1 & 1 \\ -2 & -1 & 0 & 1 & 2 \end{bmatrix}; \quad (Z'Z)^{-1} = \begin{bmatrix} 1/5 & 0 \\ 0 & 1/10 \end{bmatrix}$$

$$\hat{\underline{\beta}}_{(1)} = (Z'Z)^{-1}Z'\underline{y}_{(1)} = \begin{bmatrix} 3 \\ -.9 \end{bmatrix}; \quad \hat{\underline{\beta}}_{(2)} = (Z'Z)^{-1}Z'\underline{y}_{(2)} = \begin{bmatrix} 0 \\ 1.5 \end{bmatrix}$$

$$\hat{\beta} = \left[\hat{\underline{\beta}}_{(1)} \;\vdots\; \hat{\underline{\beta}}_{(2)}\right] = \begin{bmatrix} 3 & 0 \\ -.9 & 1.5 \end{bmatrix}$$

Hence

$$\hat{Y} = Z\hat{\beta} = \begin{bmatrix} 4.8 & -3.0 \\ 3.9 & -1.5 \\ 3.0 & 0 \\ 2.1 & 1.5 \\ 1.2 & 3.0 \end{bmatrix};$$

$$\hat{\varepsilon} = Y - \hat{Y} = \begin{bmatrix} 5 & -3 \\ 3 & -1 \\ 4 & -1 \\ 2 & 2 \\ 1 & 3 \end{bmatrix} - \begin{bmatrix} 4.8 & -3.0 \\ 3.9 & -1.5 \\ 3.0 & 0 \\ 2.1 & 1.5 \\ 1.2 & 3.0 \end{bmatrix} = \begin{bmatrix} .2 & 0 \\ -.9 & .5 \\ 1.0 & -1.0 \\ -.1 & .5 \\ -.2 & 0 \end{bmatrix}$$

$$Y'Y = \hat{Y}'\hat{Y} + \hat{\varepsilon}'\hat{\varepsilon}$$

$$\begin{bmatrix} 55 & -15 \\ -15 & 24 \end{bmatrix} = \begin{bmatrix} 53.1 & -13.5 \\ -13.5 & 22.5 \end{bmatrix} + \begin{bmatrix} 1.9 & -1.5 \\ -1.5 & 1.5 \end{bmatrix}$$

7.10 a) Using Result 7.7, the 95% confidence interval for the mean reponse is given by

$$[1,\ .5]\begin{bmatrix}3.0\\-.9\end{bmatrix} \pm 3.18\sqrt{[1,\ .5]\begin{bmatrix}.2 & 0\\0 & .1\end{bmatrix}\begin{bmatrix}1\\.5\end{bmatrix}\left(\frac{1.9}{3}\right)} \quad \text{or}$$

(1.35, 3.75).

b) Using Result 7.8, the 95% prediction interval for the actual Y is given by

$$[1,\ -.5]\begin{bmatrix}3.0\\-.9\end{bmatrix} \pm 3.18\sqrt{\left\{1 + [1,\ .5]\begin{bmatrix}.2 & 0\\0 & .1\end{bmatrix}\begin{bmatrix}1\\.5\end{bmatrix}\right\}\left(\frac{1.9}{3}\right)} \quad \text{or}$$

(-.25, 5.35) .

c) Using (7-48) a 95% prediction ellipse for the actual Y's is given by

$$[y_{01} - 2.55,\ y_{02} - .75]\begin{bmatrix}7.5 & 7.5\\7.5 & 9.5\end{bmatrix}\begin{bmatrix}y_{01} - 2.55\\y_{02} - .75\end{bmatrix}$$

$$\le (1 + .225)\left(\frac{(2)(3)}{2}\right)(19) = 69.825$$

7.12 (a) best linear predictor $= -4 + 2Z_1 - Z_2$

(b) mean square error $= \sigma_{yy} - \boldsymbol{\sigma}'_{zy}\boldsymbol{\Sigma}^{-1}_{zz}\boldsymbol{\sigma}_{zy} = 4$

(c) $$\rho_{Y(x)} = \sqrt{\frac{\boldsymbol{\sigma}'_{zy}\boldsymbol{\Sigma}^{-1}_{zz}\boldsymbol{\sigma}_{zy}}{\sigma_{yy}}} = \frac{\sqrt{5}}{3} = .745$$

(d) Following equation (7-62), we partition $\boldsymbol{\Sigma}$ as

$$\boldsymbol{\Sigma} = \left[\begin{array}{cc|c}9 & 3 & 1\\3 & 2 & 1\\\hline 1 & 1 & 1\end{array}\right]$$

and determine covariance of $\begin{bmatrix}Y\\Z_1\end{bmatrix}$ given Z_2 to be

$$\begin{bmatrix}9 & 3\\3 & 2\end{bmatrix} - \begin{bmatrix}1\\1\end{bmatrix}(1)^{-1}[1,\ 1] = \begin{bmatrix}8 & 2\\2 & 1\end{bmatrix}. \text{ Therefore}$$

$$\rho_{YZ_1 \cdot Z_2} = \frac{2}{\sqrt{8}\sqrt{1}} = \frac{\sqrt{2}}{2} = .707$$

7.14 (a) (i) The large positive correlation between a manager's experience and achieved rate of return on portfolio indicates an apparent advantage for managers with experience. (ii) The negative correlation between attitude toward risk and achieved rate of return indicates an apparent advantage for conservative managers.

(b) From (7-63)

$$r_{yz_1 \cdot z_2} = \frac{s_{yz_1 \cdot z_2}}{\sqrt{s_{yy \cdot z_2}}\sqrt{s_{z_1 z_1 \cdot z_2}}} = \frac{s_{yz_1} - \dfrac{s_{yz_2} s_{z_1 z_2}}{s_{z_2 z_2}}}{\sqrt{s_{yy} - \dfrac{s^2_{yz_2}}{s_{z_2 z_2}}}\sqrt{s_{z_1 z_1} - \dfrac{s^2_{z_1 z_2}}{s_{z_2 z_2}}}}$$

$$= \frac{r_{yz_1} - r_{yz_2} r_{z_1 z_2}}{\sqrt{1 - r^2_{yz_2}}\sqrt{1 - r^2_{z_1 z_2}}} = .31$$

Removing "years of experience" from consideration, we now have a positive correlation between "attitude toward risk" and "achieved return". After adjusting for years of experience, there is an apparent advantage to managers who take risks.

7.15 (a) MINITAB computer output gives: $\hat{y} = 11,870 + 2634z_1 + 45.2z_2$; residual sum of squares = 204995012 with 17 degrees of freedom. Thus $s = 3473$. Now for example, the estimated standard deviation of $\hat{\beta}_0$ is $\sqrt{1.99615^2} = 4906$. Similar calculations give the estimated standard deviations of $\hat{\beta}_1$ and $\hat{\beta}_2$.

(b) An analysis of the residuals indicate there are no apparent model inadequacies.

(c) The 95% prediction interval is ($51,228; $66,239)

(d) Using (7-17), $F = \frac{(45.2)(.0067)^{-1}(45.2)}{12058533} = .025$

Since $F_{1,17}(.05) = 4.45$ we cannot reject $H_0: \beta_2 = 0$. It appears as if Z_2 is not needed in the model provided Z_1 is included in the model.

7.16

Predictors	$p = r + 1$	C_p
Z_1	2	1.025
Z_2	2	12.24
Z_1, Z_2	3	3