

STA 4321 - STA5323 Exam 1

May 31, 2022

Student's name:

Book: yes no

Notes: yes no

This is a show your work, 110 minute, 300 points exam. Solutions should be submitted at the end of the exam period. There is a deduction of 15% from your score if you use your notes or your book (other than tables), and of 25% if you use them both. Two pages with formulas and NO hints of solutions of any type of exercise are Ok. You should specify on your solution file, if you used notes, book or both. You should fully justify your answers. Good luck!

EXERCISE 1. (20 points) A k -word is a sequence of k letters. How many different 11-words can be obtained if we swap at random the letters of the word TALLAHASSEE?

EXERCISE 2. (40 points) A two step experiment consists in first casting a fair die and then selecting a chip at random from one of three urns; urn I contains 3 red chips and 7 white chips, urn II contains 6 red chips and 4 white chips and urn III contains an equal number of red and blue chips. The fair die is cast and urn I is selected if 6 shows up, urn II is selected if 1 or 2 shows up, otherwise urn III is selected. Then from the selected urn a chip is drawn at random. What is the (posterior) probability that urn III has been selected if we know that a red chip was drawn?

EXERCISE 3. (35 points) Assume the r.v. X has the p.d.f. $f_X(x) = ax^2 1_{(0,2)}(x)$. Determine the value of a , and find the mean and the variance of X . Recall that $1_A(x) = 1$, if $x \in A$, and $1_A(x) = 0$, if

$x \notin A$,

EXERCISE 4. *Suppose that a deck of 52 bridge cards is dealt at random to the four players : W, N, S and E.*

a. (35 points) *What is the probability that each player will get an Ace and a Queen?*

b. (40 points) *What is the probability that each player receives 13 different face values?*

EXERCISE 5. (70 points) *One week, Toto's telephone rings 11 times, the calls being randomly distributed among the 7 days of that week. What is the probability that Toto gets at least one call each day of that week?*

EXERCISE 6. a. (30 points) *Using a conveniently selected double integral, find the numerical value of*

$$I = \int_0^{\infty} e^{-\frac{x^2}{2}} dx.$$

b. (10 points) *Determine the value of $c > 0$, to insure that $f(x) = ce^{-\frac{x^2}{2}} 1_{[0, \infty)}(x)$ is the p.d.f. of a r.v. X .*

c. (20 points) *Find the mean of X in part b.*