STA 4321- STA 5323 Final Exam

June 15, 2022

Student's name:

Book: yes no

Notes: yes no

This is a show your work, 135 minute, 400 points exam. There is a deduction of 15% from your score if you use your notes or your book (other than tables), and of 25% if you use them both. A calculator and two pages with formulas and NO hints of solutions of any type of exercise can be used at no penalty. You should fully justify your answers. Good luck!

EXERCISE 1. *(60 points) Two bullets are fired at the center of a target, which is the origin of a system of coordinates (x,y), and the coordinates of the points where the bullets hit the target are independent random vectors* $(X_1, Y_1), (X_2, Y_2)$. *For* $a = 1, 2$ *we are assuming that the marginal distributions of* X_a *and* Y_a *are independent* $\mathcal{N}(0, 2)$. *What is the probability that the distance between the two points, where the bullets hit the target is at most* 2?

EXERCISE 2. *Suppose* $X_i, i = 1, 2, 3, 4$ *are independent* $\mathcal{N}(0, 2)$ *distributed random variables. Find :*

1. (40 points) the probability that $X_1^2 + X_2^2 + X_3^2 + X_4^2 \leq 4$, *and*

b. (30 points) the approximate value of the probability that $X_1 + X_2 + X_3 + X_4 \leq 4$.

EXERCISE 3. *(70 points) A Statistics class starts at 11:30 am each of Mondays through Thursdays mornings. Each of these mornings, the professor departs from home, his departure time X having a uniform distribution, between 10:40 am and 11:05 am. It takes him between 25 and 30 minutes to get*

to class, the travel time T being also uniformly distributed. Assume X and Y are independent random variables. What is the probability that the professor arrives in class on time on Thursday?

EXERCISE 4. *(70 points) A parallel system is one that functions as long as at least one of its components functions. A particular parallel system has three independent components, the i-th component having a lifelength with and exponential(λ) distribution (i =1,2,3). The lifetime of the system is the maximum of the individual lifelengths of its components. What is the probability that lifetime of the system exceeds 2λ?*

EXERCISE 5. *(30 points) a. Show that the sum of independent $Gamma(\alpha_i, \beta), i = 1, \ldots, k$ distributed random variables has a $Gamma(\alpha, \beta)$, where $\alpha = \sum_{i=1}^{k} \alpha_i$.*

(40 points) b. The random variables X_1, X_2 ar independent, with X_1 having $Gamma(\alpha_1, \beta)$ distribution and $X_1 + X_2$ having a $Gamma(\alpha, \beta)$ distribution, $\alpha > \alpha_1$. What is the distribution of X_2?

EXERCISE 6. *(60 points) A vaccine is known to be effective in 50% of the people in which it is used. A new vaccine is tested on 100 randomly selected individuals and it is found to be effective in at least 55 of them. Evaluate the probability that this would have happened if those individuals would have been administered the standard vaccine. Which of the two vaccines is more effective? Justify your answer.*

Table of areas under the standard normal density curve from 0 to x.

x	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.00000	0.00399	0.00798	0.01197	0.01595	0.01994	0.02392	0.02790	0.03188	0.03586
0.1	0.03983	0.04380	0.04776	0.05172	0.05567	0.05962	0.06356	0.06749	0.07142	0.07535
0.2	0.07926	0.08317	0.08706	0.09095	0.09483	0.09871	0.10257	0.10642	0.11026	0.11409
0.3	0.11791	0.12172	0.12552	0.12930	0.13307	0.13683	0.14058	0.14431	0.14803	0.15173
0.4	0.15542	0.15910	0.16276	0.16640	0.17003	0.17364	0.17724	0.18082	0.18439	0.18793
0.5	0.19146	0.19497	0.19847	0.20194	0.20540	0.20884	0.21226	0.21566	0.21904	0.22240
0.6	0.22575	0.22907	0.23237	0.23565	0.23891	0.24215	0.24537	0.24857	0.25175	0.25490
0.7	0.25804	0.26115	0.26424	0.26730	0.27035	0.27337	0.27637	0.27935	0.28230	0.28524
0.8	0.28814	0.29103	0.29389	0.29673	0.29955	0.30234	0.30511	0.30785	0.31057	0.31327
0.9	0.31594	0.31859	0.32121	0.32381	0.32639	0.32894	0.33147	0.33398	0.33646	0.33891
1.0	0.34134	0.34375	0.34614	0.34849	0.35083	0.35314	0.35543	0.35769	0.35993	0.36214
1.1	0.36433	0.36650	0.36864	0.37076	0.37286	0.37493	0.37698	0.37900	0.38100	0.38298
1.2	0.38493	0.38686	0.38877	0.39065	0.39251	0.39435	0.39617	0.39796	0.39973	0.40147
1.3	0.40320	0.40490	0.40658	0.40824	0.40988	0.41149	0.41308	0.41466	0.41621	0.41774
1.4	0.41924	0.42073	0.42220	0.42364	0.42507	0.42647	0.42785	0.42922	0.43056	0.43189
1.5	0.43319	0.43448	0.43574	0.43699	0.43822	0.43943	0.44062	0.44179	0.44295	0.44408
1.6	0.44520	0.44630	0.44738	0.44845	0.44950	0.45053	0.45154	0.45254	0.45352	0.45449
1.7	0.45543	0.45637	0.45728	0.45818	0.45907	0.45994	0.46080	0.46164	0.46246	0.46327
1.8	0.46407	0.46485	0.46562	0.46638	0.46712	0.46784	0.46856	0.46926	0.46995	0.47062
1.9	0.47128	0.47193	0.47257	0.47320	0.47381	0.47441	0.47500	0.47558	0.47615	0.47670
2.0	0.47725	0.47778	0.47831	0.47882	0.47932	0.47982	0.48030	0.48077	0.48124	0.48169
2.1	0.48214	0.48257	0.48300	0.48341	0.48382	0.48422	0.48461	0.48500	0.48537	0.48574
2.2	0.48610	0.48645	0.48679	0.48713	0.48745	0.48778	0.48809	0.48840	0.48870	0.48899
2.3	0.48928	0.48956	0.48983	0.49010	0.49036	0.49061	0.49086	0.49111	0.49134	0.49158
2.4	0.49180	0.49202	0.49224	0.49245	0.49266	0.49286	0.49305	0.49324	0.49343	0.49361

x	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
2.5	0.49379	0.49396	0.49413	0.49430	0.49446	0.49461	0.49477	0.49492	0.49506	0.49520
2.6	0.49534	0.49547	0.49560	0.49573	0.49585	0.49598	0.49609	0.49621	0.49632	0.49643
2.7	0.49653	0.49664	0.49674	0.49683	0.49693	0.49702	0.49711	0.49720	0.49728	0.49736
2.8	0.49744	0.49752	0.49760	0.49767	0.49774	0.49781	0.49788	0.49795	0.49801	0.49807
2.9	0.49813	0.49819	0.49825	0.49831	0.49836	0.49841	0.49846	0.49851	0.49856	0.49861
3.0	0.49865	0.49869	0.49874	0.49878	0.49882	0.49886	0.49889	0.49893	0.49896	0.49900
3.1	0.49903	0.49906	0.49910	0.49913	0.49916	0.49918	0.49921	0.49924	0.49926	0.49929
3.2	0.49931	0.49934	0.49936	0.49938	0.49940	0.49942	0.49944	0.49946	0.49948	0.49950
3.3	0.49952	0.49953	0.49955	0.49957	0.49958	0.49960	0.49961	0.49962	0.49964	0.49965
3.4	0.49966	0.49968	0.49969	0.49970	0.49971	0.49972	0.49973	0.49974	0.49975	0.49976
3.5	0.49977	0.49978	0.49978	0.49979	0.49980	0.49981	0.49981	0.49982	0.49983	0.49983
3.6	0.49984	0.49985	0.49985	0.49986	0.49986	0.49987	0.49987	0.49988	0.49988	0.49989
3.7	0.49989	0.49990	0.49990	0.49990	0.49991	0.49991	0.49992	0.49992	0.49992	0.49992
3.8	0.49993	0.49993	0.49993	0.49994	0.49994	0.49994	0.49994	0.49995	0.49995	0.49995
3.9	0.49995	0.49995	0.49996	0.49996	0.49996	0.49996	0.49996	0.49996	0.49997	0.49997
4.0	0.49997	0.49997	0.49997	0.49997	0.49997	0.49997	0.49998	0.49998	0.49998	0.49998