STA 4322-STA 5325 Exam 1

March 24, 2022

Student's name:

Book: yes no

Notes: yes no

This is a show your work, 300 points, 70 minute exam, partially covering Chapters 5, 6 and 9 from your textbook and additional material taught this semester. There is a deduction of 15% from your score if you use your notes or your book, and of 25% if you use them both. A sheet of paper with formulas and theorems, with no hints for solutions, written on both sides is Ok. No computer access is allowed. All your answers have to be fully justified, based on definitions and theorems from the book or class notes. Good luck!

EXERCISE 1. *(40 points) Assume* $X = (X_1, \dots, X_6)^T = X_1 e_1 + \cdots + X_6 e_6$ *is a multinomial trial, where, for* $j = 1, \dots, 6$ $X_j \in \{0, 1\}, \sum_{i=1}^{6} X_i = 1$, *and the coordinate* θ_j *of the vector parameter* $\theta = (\theta_1, \dots, \theta_6)$, *is the probability for* $X = e_j$. *What is the parameter space* Θ, *of all possible values of* θ, *for this multinomial trial? What are the mean vector* $E(X)$ *and the covariance matrix* $\Sigma = Cov(X)$*?*

EXERCISE 2. *(30 points) Assume* $X_1, \dots, X_n$ *are i.i.d.r.v.'s from a normal distribution with mean* μ *and variance* μ^2. *Find a two dimensional sufficient statistic for* $\theta = (\mu, \mu^2)$.

EXERCISE 3. *(30 points) Assume* $X_1, X_2, \dots$ *is a sequence of i.i.d.r.v.'s from a Bernoulli* $B(1, 0.5)$ *distribution. What is the limiting distribution of* $n[\bar{X}_n(1 - \bar{X}_n) - \frac{1}{4}]$ *as the sample size* n *goes to* ∞ *?*

EXERCISE 4. *(a) (20 points) How are related the notions of i.i.d.r.v.'s and random sample from a probability distribution* Q? *(b)(40 points) Given i.i.d.r.v.'s* $X_1, \dots, X_n$ *from a* $\mathcal{N}(\mu, \sigma^2)$ *distribution, what is the joint density function of* $\hat{\theta} = (\bar{X}_n, S_n^2)$ *as an estimator of* $\theta = (\mu, \sigma^2) \in \mathbb{R} \times (0, \infty)$, *where* $\bar{X}_n$ *is the sample mean and* $S_n^2 = \frac{1}{n-1} \sum_{i=1}^n (X_i - \bar{X}_n)^2$ *?*

EXERCISE 5. *(a)(20 points). Give an example of an interval estimate for* μ *at confidence level* $1 - \alpha$, *that is not symmetric about* $\bar{x}_n$, *the sample mean of a random sample* $x_1, \dots, x_n$ *of size* n *from* $\mathcal{N}(\mu, 1)$. *(b)(30 points). Give an example of an interval estimate at confidence level* $1 - \alpha$ *for* σ^2 *based on a random sample* $x_1, \dots, x_n$ *from* $\mathcal{N}(\mu, \sigma^2)$.

EXERCISE 6. *(50 points) Derive a* $1 - \alpha$ *confidence level interval for the difference* $\delta = \mu_1 - \mu_2$ *of the means of two independent populations* $\mathcal{N}(\mu_a, \sigma^2), a = 1, 2$, *having the same unknown variance, based on the random samples* $x_{a,1}, \dots, x_{a,n_a}, a = 1, 2$ *from these populations.*

EXERCISE 7. *(40 points) Assume* $X_1, \dots, X_n$ *are i.i.d.r.v.'s from a normal distribution with mean* μ *and variance* 1. *A confidence interval for* μ *at level* $1 - \alpha$ *is* $(\bar{X}_n - \frac{z_{\frac{\alpha}{2}}}{\sqrt{n}}, \bar{X}_n + \frac{z_{\frac{\alpha}{2}}}{\sqrt{n}})$, *where* $z_{\frac{\alpha}{2}}$ *is defined by the property* $Pr(Z \geq z_{\frac{\alpha}{2}}) = 1 - \frac{\alpha}{2}$, *with* Z *having a standard normal distribution. Let* p *denote the probability that an additional independent random observation* X_{n+1} *having the distribution* $\mathcal{N}(\mu, 1)$ *will fall in this interval. Is* p *greater than,, less than, or equal to* $1 - \alpha$? *Prove your answer.*