STA 5707 - STA 4702 - Exam 1

November 16, 2022

Student's name:

Notes: yes no

Book: yes no

This is 300 points "show your work", 70 minute exam. You are free to use the tables at the back of the textbook. There is a deduction of 15% from your score if you use your notes or your book (other than tables), and of 25% if you use them both. Two pages with formulas and NO hints of solutions of any type of exercise are ok. You should fully justify your answers. Good luck!

EXERCISE 1. *(40 points) Assume* $M \sim W_m(\cdot|\Sigma)$, *and* A *is a random vector such that* $A \neq 0_m, A \sqcup M$. *What is the distribution of* $\frac{A^T M A}{A^T \Sigma A}$?

EXERCISE 2. *(40 points) Assume* $x_1, x_2, \ldots, x_n$ *is a sample from a multivariate normal distribution in* $\mathbb{R}^m$, *having a nonsingular sample covariance matrix. Consider the transformation* $Y = h(X) = AX$, *where* $A^T A = pI_p$, *with* I_p *the identity matrix. Show the the value of the* T^2 *statistic for the transformed data set* $y_1, y_2, \ldots, y_n$, *where* $y_i = h(x_i), i = 1, \ldots, n$, *is the same as the value of this statistic for* $x_1, x_2, \ldots, x_n$.

EXERCISE 3. *(40 points) Assume the data matrix* $\mathbf{x}$ *for a random sample of size* $n = 4$ *from a bivariate normal population of mean vector* $\mu_0 = \binom{7}{7}$ *is given by* $\mathbf{x} = \begin{pmatrix} 6 & 10 \\ 5 & 8 \\ 9 & 7 \\ 4 & 3 \end{pmatrix}$. *Is the test* $\mu = \mu_0$ *statistically significant?*

EXERCISE 4. *(40 points) Assume* X *is a* 4×4 *sampling matrix from* $\mathcal{N}_4(0_4, \begin{pmatrix} 4 & 1 & 1 & 0 \\ 1 & 4 & 0 & 1 \\ 1 & 0 & 4 & 1 \\ 0 & 1 & 1 & 4 \end{pmatrix})$, *where* C *is the symmetric matrix* $C = \frac{1}{4} \begin{pmatrix} 1 & -1 & 1 & -1 \\ -1 & 1 & -1 & 1 \\ 1 & -1 & 1 & -1 \\ -1 & 1 & -1 & 1 \end{pmatrix}$. *What is the distribution of* XCX^T?

EXERCISE 5. *(50 points) Consider the covariance matrix of the random vector* $X = (X_1\ X_2\ X_3\ X_4\ X_5\ X_6)^T$, *given by*

$$\Sigma = \sigma^2 \begin{pmatrix} 1 & \rho & \rho & \rho & \rho & \rho \\ \rho & 1 & \rho & \rho & \rho & \rho \\ \rho & \rho & 1 & \rho & \rho & \rho \\ \rho & \rho & \rho & 1 & \rho & \rho \\ \rho & \rho & \rho & \rho & 1 & \rho \\ \rho & \rho & \rho & \rho & \rho & 1 \end{pmatrix} \tag{1}$$

and assume X_1 *and* X_3 *are positively the correlated. Determine proportion of the total population variance explained by the largest principal component.*

EXERCISE 6. *Observations on two responses are collected for three treatments. The observed vectors* $\begin{pmatrix} x_1 \\ x_2 \end{pmatrix}$ *are*

Treatment 1: $\begin{pmatrix} 6 \\ 7 \end{pmatrix} \begin{pmatrix} 7 \\ 9 \end{pmatrix} \begin{pmatrix} 4 \\ 9 \end{pmatrix} \begin{pmatrix} 8 \\ 6 \end{pmatrix} \begin{pmatrix} 5 \\ 9 \end{pmatrix}$

Treatment 2: $\begin{pmatrix} 2 \\ 3 \end{pmatrix} \begin{pmatrix} 3 \\ 1 \end{pmatrix} \begin{pmatrix} 5 \\ 1 \end{pmatrix} \begin{pmatrix} 2 \\ 3 \end{pmatrix}$

Treatment 3: $\begin{pmatrix} 3 \\ 3 \end{pmatrix} \begin{pmatrix} 2 \\ 3 \end{pmatrix} \begin{pmatrix} 1 \\ 6 \end{pmatrix}$

a. (60 points) Evaluate Wilks lambda statistic Λ^*

b. (30 points) Test the equality of the mean vectors in the three groups al significance level $\alpha = 0.01$. *What are your assumptions?*