STA5707-STA4702- Exam

November 22, 2022

Exam 1

Student's name:

Notes: yes no

This is a show your work, 70 minute exam. There is a deduction of 15% from your score if you use your notes or your book, and of 25% if you use them both. A one page formula sheet with NO hints of solutions of types of exercise, and tables only from the back of the textbook are ok. You should fully justify your answers. Good luck!

EXERCISE 1. *Assume* $X \sim N_3(\mu, \Sigma)$, *where* $\mu^T = [2, -3, 1]$ *and* $\Sigma = \begin{pmatrix} 1 & 1 & 1 \\ 1 & 3 & 2 \\ 1 & 2 & 2 \end{pmatrix}$.

a. (20 points) Find the distribution of $2X^1 - 3X^2 + X^3$.

b. (30 points) Relabel the variables if necessary, and find a 2×1 *vector* **a** *such that* X_2 *and* $X^2 - [X^1 X^3]\boldsymbol{a}$ *are independent.*

c. (40 points) Specify the conditional distribution of X^1 *given* $X^2 = x^2$ *and* $X^3 = x^3$.

EXERCISE 2. *Let* $X_1, \ldots, X_{60}$ *be a random sample of size 60 from an* $N_5(\mu, \Sigma)$ *population, where* Σ *is given by*

$$\Sigma = \begin{pmatrix} 4 & 1 & 1 & 0 & 0 \\ 1 & 4 & 0 & 1 & 0 \\ 1 & 0 & 4 & 1 & 0 \\ 0 & 1 & 1 & 4 & 0 \\ 0 & 0 & 0 & 0 & 2 \end{pmatrix} \tag{1}$$

Specify each of the following distributions:

a. (10 points) The distribution of $(X_1 - \mu)^T \Sigma^{-1} (X_1 - \mu)$

b. (15 points) The distributions of $\overline{X}$ *and* $\sqrt{60}(\overline{X} - \mu)$

c. (15 points) The distribution of $59 S_u$

d. (20 points) The distribution of $59 B S_u B^T$, *where* $B = \begin{pmatrix} 1 & -1 & -1 & 0 & 0 \\ 0 & 0 & 0 & -1 & -1 \end{pmatrix}$.

e.(10 points) $E(S_u)$.

f. (20 points) The exact distribution of $60(\overline{X} - \mu)^T S_u^{-1} (\overline{X} - \mu)$

EXERCISE 3. *(50 points) Assume* $M \sim W_m(\cdot|\Sigma)$, *and* A *is a random vector such that* $A^T M A \neq 0, A \sqcup M$. *Show that* $\frac{A^T M A}{A^T \Sigma A} \tilde{-} \chi^2_m$.

EXERCISE 4. *Observations on two responses are collected for two treatments. The observed vectors* $\begin{pmatrix} x_1 \\ x_2 \end{pmatrix}$ *are*

Treatment 1: $\begin{pmatrix} -6 \\ -7 \end{pmatrix} \begin{pmatrix} -7 \\ -9 \end{pmatrix} \begin{pmatrix} -4 \\ -9 \end{pmatrix} \begin{pmatrix} -8 \\ -6 \end{pmatrix} \begin{pmatrix} -5 \\ -9 \end{pmatrix}$

Treatment 2: $\begin{pmatrix} -2 \\ -3 \end{pmatrix} \begin{pmatrix} -3 \\ -1 \end{pmatrix} \begin{pmatrix} -5 \\ -1 \end{pmatrix} \begin{pmatrix} -2 \\ -3 \end{pmatrix}$

a. (50 points) Is there any significant difference between the mean treatments?

b. (20 points) What are the assumptions justifying your answer to b.?